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Quantum teleportation of a continuous-variable state is studied for the quantum channel of a two-mode squeezed vacuum influenced by a thermal environment. Each mode of the squeezed vacuum is assumed to undergo the same thermal influence. It is found that when the mixed two-mode squeezed vacuum for the quantum channel is separable, any nonclassical features, which may be imposed in an original unknown state, cannot be transferred to a receiving station. A two-mode Gaussian state, one of which is a mixed two-mode squeezed vacuum, is separable if and only if a positive well-defined P function can be assigned to it. The fidelity of teleportation is considered in terms of the noise factor given by the imperfect channel. It is found that quantum teleportation may give more noise than direct transmission of a field under the thermal environment, which is due to the fragile nature of quantum entanglement of the quantum channel.

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I. INTRODUCTION

Quantum teleportation is one of the important manifestations of quantum mechanics. By quantum teleportation an unknown quantum state is destroyed at a sending station while its replica state appears at a remote receiving station via dual quantum and classical channels. The key to quantum teleportation is the entanglement of the quantum channel. Quantum teleportation has been studied for various systems including two-level systems [1], N-dimensional systems [2], and continuous variables [3–5]. In particular, quantum teleportation of continuous variable states has been at a focus because of a high detection efficiency and handy manipulation of continuous variable states [4,6].

Quantum teleportation of a continuous-variable state was first suggested by Vaidman employing the Einstein-Podolsky-Rosen (EPR) state [7] for the quantum channel in the framework of nonlocal measurements [3]. Braunstein and Kimble made a use of quadrature-phase entanglement in a two-mode squeezed vacuum to teleport the quadrature-phase variables. With the high detection efficiency of the homodyne measurement and highly squeezed light, Ralph and Lam [5] and Furusawa *et al.* [6] realized quantum teleportation of continuous variable states by experiments. Ralph and Lam produced the required entangled state using two bright squeezed sources. A two-mode squeezed vacuum is entangled with respect not only to quadrature phases but also to photon-number difference and phase sum. Based on this number-phase entanglement, Milburn and Braunstein suggested another protocol to teleport a continuous variable state [8].

There are a few problems in the quantum teleportation of quadrature-phase variables using the two-mode squeezed vacuum. The perfect quantum teleportation is possible only with a maximally entangled state which means infinite squeezing in the squeezed state. The mean energy of a two-mode squeezed state increases exponentially as the squeezing increases so that the maximally entangled squeezed state is unphysical. As the quantum channel is exposed to the real world, it is influenced by the environment, which turns the *pure* squeezed state into a *mixture* and deteriorates the entanglement property. The environmental effect is unavoidable for any type of teleportation and there have been suggestions to purify mixed entangled state into a maximally entangled singlet state for a discrete two-level system [9]. Duan *et al.* suggested a way to purify a Gaussian continuous variable state [10]. However, their purification protocol may concentrate entanglement only to a finite dimensional Hilbert space. In fact, it is impossible to purify a two-mode squeezed state into a maximally entangled state as it is unphysical. Opartny *et al.* showed that the problem of not having the maximally entangled squeezed vacuum can be overcome by conditional measurements [11]. Entanglement quantification and purification for continuous-variable states has been studied by Parker *et al.* [12]. The imperfect detection efficiency and the imperfect realization of unitary transformation at the receiving station can also lower the efficiency of teleportation.

In this paper, we are interested in the efficiency of quantum teleportation in the real world. Nonclassical properties such as sub-Poissonicity and squeezing of the original state can be very useful for communication purposes. As the quantum channel is not maximally entangled, some or all of the nonclassical properties can be lost during the teleportation. Braunstein and Kimble found that when the quantum channel is not squeezed, i.e., when the channel is merely a two-mode vacuum, no quantum features can be observed in the teleported state [4]. This is due to quantum tariffs of vacuum noise, which arises at the sending and receiving stations. The tariff was coined as *quduty* by Braunstein and Kimble. The pure two-mode squeezed state becomes mixed as the quantum channel is embedded in the environment. Quantum teleportation via the mixed channel can bear a different nature. For example, one may ask "Does the classical correlation play any role to transfer the nonclassical features?" It is not clear so far under which condition any nonclassical features implicit in an original unknown state cannot be transferred by teleportation via a mixed channel. We also consider the fidelity of teleportation to measure how close the teleported state is to the original state when the quantum channel is mixed. Popescu studied quantum teleportation of a discrete two-level system for a mixed quantum channel and found that even when the quantum channel is not maximally entangled, it has the fidelity better than any classical teleportation protocol [13]. In this paper, we restrict ourselves to the situation that the decoherence effect is the same on each mode of the two-mode squeezed vacuum.

The continuous variable state can be easily analyzed using the quasiprobability functions [14]. The description of a quantum-mechanical state in phase space is not unique due to the uncertainty principle; hence there are a family of quasiprobability functions of which the P, Q, and Wigner functions are widely used [15]. In particular, it is well-known that the P function can be used as a measure of the nonclassicality of a given field because the P function is positive well-defined only for a classical state [16]. The nonclassical depth is defined based on how much noise to put into the nonclassical state to have a positive well-defined P function.

When teleportation is imperfect, a noise-added replica state is produced at the receiving station. By analyzing the added noise, we find the critical point for the quantum channel not to transfer any nonclassical features which may be imposed in an original unknown state. We examine the coincidence of the critical point with the moment when the quantum channel becomes separable. To do that we find the necessary and sufficient condition of separability for any two-mode Gaussian state [17], one of which is the mixed two-mode squeezed state. The fidelity, which is defined as the inner product of the original and teleported states, can be represented by the overlap of their Wigner functions [18]. We show that the fidelity is a function of the added noise.

The added noise by teleportation is compared with that by direct transmission of the original state. It is found that the nonclassical nature of the original state can be more easily lost by teleportation than by direct transmission. This is because teleportation relies on the entanglement of the quantum channel, which is very fragile.

II. QUASIPROBABILITY FUNCTIONS

Before considering quantum teleportation, we briefly introduce the quasiprobability functions. The family of quasiprobability functions are obtained by the following convolution relation

$$R_\sigma(\alpha) = \int d^2\beta \left[\frac{2}{\pi(1-\sigma)} \exp\left(-\frac{2\left|\alpha-\beta\right|^2}{1-\sigma}\right)\right] P(\beta) \tag{1}$$

where the σ-parameterized $R_\sigma(\alpha)$ function becomes Q function for $\sigma = -1$, Wigner (W) function for $\sigma = 0$, and P function for $\sigma = 1$. By the Fourier transform, we find the relation between their characteristic functions

$$C^R_\sigma(\xi) = \exp\left[-\frac{(1-\sigma)|\xi|^2}{2}\right] C^P(\xi) \tag{2}$$

where $C^R_\sigma(\xi)$ and $C^P_\sigma(\xi)$ are the characteristic functions for the R and P functions, respectively. The family of two-mode quasiprobability functions can be analogously defined as

$$R_\sigma(\alpha,\beta) = \frac{4}{\pi^2(1-\sigma)^2}\int d^2\phi d^2\eta \times \exp\left(-\frac{2\left|\alpha-\phi\right|^2}{1-\sigma} - \frac{2\left|\beta-\eta\right|^2}{1-\sigma}\right) P(\phi,\eta). \tag{3}$$

III. TELEPORTATION FOR CONTINUOUS VARIABLES IN THERMAL ENVIRONMENTS

A continuous variable state $\hat\rho_o$ can be teleported with use of a two-mode squeezed vacuum for a quantum channel [4]. Two modes b and c of the squeezed vacuum are distributed separately to a sending and a receiving stations. The protocol comprises two operations at the sending station and one operation at the receiving station. At the sending station, the original unknown state of mode a is mixed with a mode b of the quantum channel by a 50/50 beam splitter. Two conjugate quadrature variables are measured respectively for the two output fields of the beam splitter. The measurement results are sent to the receiving station through the classical channel. The other mode c of the squeezed vacuum is then displaced at the receiving station according to the measurement results. It is important to displace the photon of mode c entangled with the photon measured at the sending station. Braunstein and Kimble considered the teleportation protocol for the *pure* state of the quantum channel [4]. In this paper we investigate the teleportation via the mixed quantum channel to consider the influence of a thermal environment. We assume that the thermal environment gives the same effect on each mode of the quantum channel and the original state is prepared in a pure state.

The two-mode squeezed vacuum of the quantum channel is entangled and represented by the Wigner function [19]

$$W_{qc}(\alpha_b, \alpha_c) = \frac{4}{\pi^2} \exp\left[-2\left(|\alpha_b|^2 + |\alpha_c|^2\right)\cosh 2s_{qc} \right.$$
$$\left. +2\left(\alpha_b\alpha_c + \alpha_b^*\alpha_c^*\right)\sinh 2s_{qc}\right], \qquad (4)$$

where s_{qc} is the degree of squeezing and the complex quadrature phase variable $\alpha_{b,c} = \alpha_{b,c}^r + i\alpha_{b,c}^i$. When $s_{qc} \to \infty$ the state (4) manifests the maximum entanglement and becomes an EPR state. However, the mean photon number, which is $2\sinh^2 s_{sq}$, becomes infinity in this limit.

Before the action of the beam splitter, the total state is a product of the original state and the state of the quantum channel, which is represented by the total Wigner function $W_t(\alpha_a, \alpha_b, \alpha_c) = W_o(\alpha_a)W_{qc}(\alpha_b, \alpha_c)$ where $W_o(\alpha_a)$ is the Wigner function of the original state $\hat{\rho}_o$. The product state of the original field and quantum channel becomes entangled at the beam splitter. Considering the unitary action of the beam splitter, the quadrature variables $\alpha_{d,e}$ of the output fields are related to those of the input fields: $\alpha_{d,e} = (\alpha_b \pm \alpha_a)/\sqrt{2}$. The Wigner function $W_t^B(\alpha_d, \alpha_e, \alpha_c)$ for the total field after the beam splitter is

$$W_t^B(\alpha_d, \alpha_e, \alpha_c) = W_t\left(\frac{\alpha_e + \alpha_d}{\sqrt{2}}, \frac{\alpha_e - \alpha_d}{\sqrt{2}}, \alpha_c\right) \qquad (5)$$

which exhibits entanglement between the modes a and b.

Setting homodyne detectors at the output ports of the beam splitter, the imaginary part of α_d and the real part of α_e are simultaneously measured by appropriately choosing the phases of reference fields for the homodyne detectors. Each measurement result is transmitted to the receiving station to displace the quadrature variables of the field mode c. We have to make it sure that the displacement operation is done on the photon of mode c entangled with the photon measured at the sending station. After the displacement $\Delta(\alpha_d^i, \alpha_e^r)$ the field of mode c becomes to be represented by the Wigner function $W_r(\alpha_c)$;

$$W_r(\alpha_c) = \int d^2\alpha_d\, d^2\alpha_e W_t^B(\alpha_d, \alpha_e, \alpha_c - \Delta(\alpha_d^i, \alpha_e^r)). \quad (6)$$

Braunstein and Kimble [4] found that the displacement of $\Delta(\alpha_d^i, \alpha_e^r) = -\sqrt{2}(\alpha_e^r - i\alpha_d^i)$ maximizes the fidelity when the channel is a pure two-mode squeezed state. The probability $P(\alpha_d^i, \alpha_e^r)$ of measuring α_d^i and α_e^r at the sending station is the same as the marginal Wigner function

$$P(\alpha_d^i, \alpha_e^r) = \int d\alpha_d^r\, d\alpha_e^i d^2\alpha_c W_t^B(\alpha_d, \alpha_e, \alpha_c). \qquad (7)$$

A. two-mode squeezed vacuum in thermal environments

The quantum channel initially in the two-mode squeezed vacuum results in a mixed state by the interaction with the thermal environment. Assuming that two thermal modes are independently coupled with the quantum channel the dynamics of the squeezed field is described by a Fokker-Planck equation in the interaction picture [20]

$$\frac{\partial W_{qc}(\alpha_b, \alpha_c; t)}{\partial t} = \frac{\gamma}{2}\sum_{i=b,c}\left[\frac{\partial}{\partial\alpha_i}\alpha_i + \frac{\partial}{\partial\alpha_i^*}\alpha_i^* \right.$$
$$\left. + (1+2\bar{n})\frac{\partial^2}{\partial\alpha_i\partial\alpha_i^*}\right] W_{qc}(\alpha_b, \alpha_c; t), \quad (8)$$

where $\bar{n}$ is the average photon number of the thermal environment. The two thermal modes are assumed to have the same average energy and coupled with the channel in the same strength. This assumption is reasonable as the two modes of the squeezed state are in the same frequency and the temperature of the environment is normally the same. By solving the Fokker-Planck equation (8), the time-dependent Wigner function is obtained as [20]

$$W_{qc}(\alpha_b, \alpha_c; T) = \mathcal{N}\exp\left[-\frac{2\Gamma}{\Gamma^2 - \Lambda^2}\left(|\alpha_b|^2 + |\alpha_c|^2\right) \right.$$
$$\left. + \frac{2\Lambda}{\Gamma^2 - \Lambda^2}\left(\alpha_b\alpha_c + \alpha_b^*\alpha_c^*\right)\right] \quad (9)$$

where $\mathcal{N}$ is the normalization factor and two parameters, $\Gamma = T(1+2\bar{n}) + (1-T)\cosh 2s_{qc}$, $\Lambda = (1-T)\sinh 2s_{qc}$. The renormalized time $T(t) = 1 - \exp(-\gamma t)$. The relative strength of Λ to Γ determines how much the mixed channel is entangled. When Λ is zero for $T \to 1$, the channel loses any correlation so to have neither classical nor quantum correlation. At $T = 0$ the mixed squeezed state (9) is simply the squeezed vacuum (4).

When the quantum channel is embedded in thermal environments, the teleported state is still represented by the Wigner function (6) with the quantum channel (9). However, a question remains in the unitary operation at the receiving station when the channel is a mixed state. According to the philosophy of the faithful teleportation, the displacement has to be determined to maximize the fidelity of teleportation. The fidelity $\mathcal{F}$, which measures how close the teleported state is to the original state, is the projection of the original pure state $|\Psi_o\rangle$ onto the teleported state of the density operator $\hat{\rho}_r$: $\mathcal{F} = \langle\Psi_o|\hat{\rho}_r|\Psi_o\rangle$. The fidelity is also represented by the overlap between the Wigner functions for the original and teleported states [18];

$$\mathcal{F} = \pi\int d^2\alpha W_o(\alpha)W_r(\alpha). \qquad (10)$$

For a maximally entangled quantum channel, the original pure state is reproduced at the receiving station and the fidelity is unity. For an impure or partially entangled channel, the unitary operation at the receiving station may depend on original states to maximize the fidelity, which has been shown for the teleportation of a two-level state [13,21]. For the infinite dimensional Hilbert space, a formal study is very much complicated. However, we have found that even when the channel is mixed, the displacement of $\Delta(\alpha_d^i, \alpha_e^r) = -\sqrt{2}(\alpha_e^r - i\alpha_d^i)$ maximizes the fidelity for a coherent projector $|\mu\rangle\langle\nu^*|$, where $|\mu\rangle$ and $|\nu^*\rangle$ are coherent-state bases. An unknown state can be written as a weighted integral of the coherent projection operators

$$\hat{\rho}_o = \int d^2\mu d^2\nu P_o(\mu,\nu)|\mu\rangle\langle\nu^*| \tag{11}$$

where $P(\mu,\nu)$ is proportional to the positive-P function [22]. The unitary operation, which maximizes the fidelity, at the receiving station is thus independent of the original state.

B. separability of the quantum channel

A discrete bipartite system of modes b and c is separable when its density operator is represented by $\hat{\rho} = \sum_r P_r \hat{\rho}_{b,r} \otimes \hat{\rho}_{c,r}$. Separability and measures of entanglement for continuous variable states has been studied [12,17]. In particular, Duan *et al.* found a criterion to determine separability of a two-mode Gaussian state. Here, we have a somewhat different approach to find when a two-mode squeezed vacuum in thermal environments is separable and not quantum-mechanically entangled. Our analysis of separability for the mixed squeezed vacuum is extended and fully described for any two-mode Gaussian state in Appendix.

As shown in Appendix, the mixed two-mode squeezed vacuum in the thermal environment is separable when a positive definite P function can be assigned to it. The mixed two-mode squeezed vacuum serving the quantum channel can then be written by a statistical mixture of the direct-product states;

$$\hat{\rho}_{qc} = \int d^2\beta \mathcal{P}(\beta)\hat{\rho}_b(\beta) \otimes \hat{\rho}_c(\beta) \tag{12}$$

where $\mathcal{P}(\beta)$ is a probability density function.

With use of Eqs. (3) and (9), we find that the mixed two-mode squeezed vacuum is separable when $n_\tau = 1$ where n_τ is defined as

$$\begin{aligned} n_\tau(\bar{n}, s_{qc}, T) &\equiv \Gamma - \Lambda \\ &= (2\bar{n}+1)\,T + (1-T)\exp(-2s_{qc}) \end{aligned} \tag{13}$$

according to the condition (A10). This is in agreement with Duan *et al.*'s separation criterion [17]. The pure two-mode squeezed vacuum for $T = 0$, is never separable unless $s_{qc} = 0$. For the zero temperature environment, i.e., $\bar{n} = 0$, the two-mode squeezed state stays quantum-mechanically entangled at any time. For the reasons given in Sec. IV, we call n_τ as the noise factor.

If $n_\tau < 1$, the quantum channel state is entangled and the teleportation is performed at the quantum level. When $n_\tau \geq 1$, the quantum channel is no longer quantum-mechanically entangled. However the inter-mode correlation is still there as $\Lambda \neq 0$ in Eq. (9). Questions then arise: Does this classical correlation influence the teleportation? Can any nonclassical properties imposed in an original state be teleported by the classically-correlated channel? Braunstein and Kimble found that when a pure two-mode squeezed state is separable, i.e., $s_{sq} = 0$, observation of any nonclassical features in the teleported state is precluded. However, when a pure state is separable there is no classical correlation either.

IV. TRANSFER OF NONCLASSICAL FEATURES

An imperfect replica state is reproduced at the receiving station when the quantum channel is not maximally entangled. It is well known that any linear noise-addition process, for example linear dissipation and amplification, can be described by the convolution relation of the quasiprobability functions [23]. With use of the Wigner functions for an arbitrary original state (11) and for an impure quantum channel (9), we find that the equation (6) leads to the following convolution relation

$$W_r(\alpha) = \int d^2\beta P_\tau(\alpha-\beta)W_o(\beta) \tag{14}$$

where the function P_τ characterizes the teleportation process;

$$P_\tau(\alpha-\beta) = \frac{1}{\pi n_\tau}\exp\left(-\frac{1}{n_\tau}|\alpha-\beta|^2\right) \tag{15}$$

and the noise factor n_τ, defined in Eq. (13), is completely determined by the characteristics of the quantum channel. The noise factor increases monotonously as the interaction time T increases. The larger the initial squeezing, the less vulnerable the quantum channel is.

The noise factor n_τ is related to the fidelity. With use of Eqs. (10) and (14) the fidelity can be written as

$$\mathcal{F} = \pi\int d^2\alpha d^2\beta W_o(\alpha)P_\tau(\alpha-\beta)W_o(\beta). \tag{16}$$

In the limit of $n_\tau \to 0$, the function $P_\tau(\alpha-\beta)$ in Eq. (15) becomes a delta function and the fidelity becomes unity. The teleportation loses the original information completely with $\mathcal{F} = 0$ in the limit of $n_\tau \to \infty$.

The properties of the nonclassical states have been calculated and illustrated by quasiprobability functions. The nonclassical features are associated especially with negative values and singularity of the quasiprobability P function [16,24,25]. Suppose an original state whose P

function is not positive everywhere in phase space. When this state is teleported, its nonclassical features are certainly transferred to the teleported state if the teleportation is perfect. If the teleportation is poor, the teleported state may have its P function positive definite and lose the nonclassical features.

By the Fourier transform of Eq. (14), the convolution relation is represented in terms of the characteristic functions as

$$C_r^W(\xi) = \exp(-\bar{n}_\tau|\xi|^2)C_o^W(\xi). \tag{17}$$

Using the relation (2) between characteristic functions, Eq.(17) is written as

$$C_r^P(\xi) = \exp[-(n_\tau - 1)|\xi|^2]C_o^Q(\xi), \tag{18}$$

where $C_o^Q(\xi)$ is the characteristic function for $R_{\sigma=-1}(\alpha)$ of the original state. The P function is not semi-positive definite if its characteristic function $C_r^P(\xi)$ is not inverse-Fourier-transformable. Even when it is inverse-Fourier-transformable, there is a chance for the P function to become negative at some points of phase space. Lütkenhaus and Barnett found that only when $\sigma \le -1$ the quasiprobability $R_\sigma(\alpha)$ for any state is semi-positive definite. We are sure that, for any original state, the left-hand side of Eq. (18) is inverse-Fourier transformed to a P function semi-positive definite only when $n_\tau \ge 1$. This condition is the same as the separability condition (13) for the quantum channel. We conclude that *when a quantum channel is separable, i.e., not quantum-mechanically entangled, no nonclassical features implicit in an original state is transferred by teleportation.* In other words, nonclassical features are not teleported via a classically-correlated channel.

There are two well-known nonclassical properties which a continuous-variable state may have: Sub-Poissonian photon statistics and quadrature squeezing. The two nonclassical properties have been studied for noiseless communication. We analyze the transfer of these properties by teleportation in the following subsections.

A. sub-Poissonian statistics and Fock state

A state is defined to be sub-Poissonian when its photon-number variance $(\Delta N)^2$ is smaller than its mean photon number $\bar{N}$. The expectation value of an observable for a state can be obtained by use of the characteristic function $C^P(\xi)$ for its P function [15];

$$\langle(\hat{a}^\dagger)^m\hat{a}^n\rangle = \frac{\partial^m}{\partial\xi^m}\frac{\partial^n}{\partial(-\xi^*)^n}C^P(\xi)\bigg|_{\xi=\xi^*=0}. \tag{19}$$

Substituting Eq. (18) into Eq. (19), we find that the teleported state is sub-Poissonian when the noise factor,

$$n_\tau < \sqrt{\bar{N}_o^2 + \bar{N}_o - (\Delta N_o)^2} - \bar{N}_o. \tag{20}$$

where $\bar{N}_o$ and $(\Delta N_o)^2$ are the mean photon number and photon-number variance for the original state. If the original state is Poissonian or super-Poissonian, the right-hand side of the inequality is either negative or imaginary so the teleported state is never sub-Poissonian.

Assuming the largest sub-Poissonicity, $(\Delta N_o)^2 = 0$, for the original state, it is found that when the noise factor $n_\tau < \sqrt{\bar{N}_o^2 + \bar{N}_o} - \bar{N} \le 1/2$, some sub-Poissonian property is found in the teleported state. Thus, if the noise factor of the quantum channel is larger than or equal to $1/2$, the transfer of any sub-Poissonian property is precluded.

A Fock state $|m\rangle$ has a definite energy and its photon-number variance is zero. When this extreme sub-Poissonian field is teleported, the mean photon number and mean variance are $\bar{N}_r = m + n_\tau$ and $\Delta N_r^2 = (2m+1)n_\tau + n_\tau^2$ at the receiving station. The Fock state $|m\rangle$ is written in the Wigner representation as

$$W_o(\alpha, m) = \frac{2}{\pi}(-1)^m \exp\left(-2|\alpha|^2\right) L_m\left(4|\alpha|^2\right). \tag{21}$$

where L_m is a Laguerre polynomial. From the convolution relation (14), the teleported state is obtained as

$$W_r(\alpha) = \frac{2}{\pi}\frac{(2n_\tau - 1)^m}{(2n_\tau + 1)^{m+1}}\exp\left(-\frac{2|\alpha|^2}{2n_\tau + 1}\right) \times L_m\left(-\frac{4|\alpha|^2}{(2n_\tau)^2 - 1}\right). \tag{22}$$

The fidelity for the Fock state is given by Eq. (16);

$$\mathcal{F}_m = \frac{(1 - n_\tau)^m}{(1 + n_\tau)^{m+1}}P_m\left(\frac{1 + n_\tau^2}{1 - n_\tau^2}\right) \tag{23}$$

where P_m is a Legendre polynomial. When $n_\tau = 0$, $\mathcal{F}_m = 1$. In the limit of $n_\tau = 1$, where the teleportation is classical, the fidelity $\mathcal{F}_m = (1/4)^m$ for $m \neq 0$. The vacuum state has the fidelity $\mathcal{F}_0 = 1/2$ in the limit.

B. quadrature squeezing and squeezed state

We examine the transfer of quadrature squeezing which an unknown original state may have. The quadrature-phase operator is defined as

$$\hat{X}(\phi) = e^{-i\phi}\hat{a} + e^{i\phi}\hat{a}^\dagger \tag{24}$$

where $\hat{a}$ ($\hat{a}^\dagger$) is an annihilation (creation) operator and ϕ related to the angle in phase space. A state is said to be squeezed if the quadrature-phase variance $[\Delta X(\phi)]^2 < 1$ for an angle ϕ. Substituting Eq. (18) into Eq. (19), the mean quadrature phase $\bar{X}(\phi)$ and variance $[\Delta X(\phi)]^2$ can be calculated

$$\bar{X}_r(\phi) = \bar{X}_o(\phi); \quad [\Delta X_r(\phi)]^2 = [\Delta X_o(\phi)]^2 + 2n_\tau, \tag{25}$$

where $\bar{X}_o(\phi)$ and $[\Delta X_o(\phi)]^2$ are the mean quadrature phase and variance for the original state. It is interesting to realize that *the mean quadrature phase does not change at all during teleportation*. This property holds regardless of the channel entanglement.

The teleported state exhibits quadrature squeezing if

$$n_\tau < \frac{1}{2}\left\{1-[\Delta X_u^2(\phi)]^2\right\} \leq \frac{1}{2}. \tag{26}$$

Suppose that the original state has the absolute minimum variance $[\Delta X_o(\phi')]^2 = 0$ at $\phi = \phi'$. Then its teleported state is also squeezed if the quantum channel is entangled enough to be represented by the noise factor $n_\tau < 1/2$. We note that the condition $n_\tau < 1/2$ applies to the survival of both quadrature squeezing and sub-Poissonian statistics.

A squeezed vacuum with the degree of squeezing s_o is written in the Wigner representation as

$$W_o(\alpha) = \frac{2}{\pi}\exp\left[-2\exp(2s_o)\alpha_r^2 - 2\exp(-2s_o)\alpha_i^2\right] \tag{27}$$

where α_r and α_i are real and imaginary parts of α. Its teleported state is represented by the Wigner function

$$W_r(\alpha) = \frac{2}{\pi\sqrt{A(s_o)A(-s_o)}}\exp\left[-\frac{2}{A(s_o)}\alpha_r^2 - \frac{2}{A(-s_o)}\alpha_i^2\right] \tag{28}$$

where the parameter $A(s_o) = 2n_\tau + \exp(-2s_o)$. The fidelity is given by

$$\mathcal{F}(s_o) = \left(n_\tau^2 + 2n_\tau\cosh 2s_o + 1\right)^{-1/2}. \tag{29}$$

When the teleportation is classical with $n_\tau = 1$, $\mathcal{F}(s_o) = (2+2\cosh 2s_o)^{-1/2}$.

V. REMARKS

Quantum teleportation can be made more reliable by sophisticated schemes such as purification of the impure or partially entangled quantum channel [12,10], detection with perfect efficiency, and well-defined unitary operation. However, in the real world, the influence of noise cannot easily be disregarded. We have been interested in the influence of noise on the transfer of nonclassicalities which may be imposed in an original unknown state. To make the problem simple while honoring the real experimental situation, we assumed that the same amount of noise affects the two modes of the quantum channel. We found that when the quantum channel is separable the transfer of any nonclassicality is impossible: Nonclassical features can not be teleported via a classically-correlated channel. The separability of a two-mode Gaussian state is considered using the possibility of assigning a positive well-defined P function to the state after some local unitary operations. We have analyzed the transfer of well-known nonclassical features such as sub-Poissonicity and quadrature squeezing. The teleportation of the two nonclassical features is ruled out under the same noise level. The faithfulness of the teleportation has also been discussed and the fidelities have been found for the initial Fock state and squeezed state. Because one of the important ingredients of teleportation is that the original state is *unknown* at the sending station. Thus our measure of noise factor n_τ, which depends only on the quality of the channel, is important. Of course, to represent the quality of the teleportation by a fidelity we have to know the average fidelity for the teleportation, which is under investigation.

One question still arises: Is the teleportation better than the direct transmission to transfer a nonclassical field? A field may be deteriorated by the thermal environment during the direct transmission. Solving a similar Fokker-Planck equation to Eq. (8) for a *single-mode* field, we find that, by the direct transmission, the Wigner function at the receiving station can be represented by the same equation as Eq. (14) with the different noise factor n_d [23]:

$$n_d = \bar{n}T. \tag{30}$$

Assuming that the imperfect teleportation is caused only by the impure quantum channel embedded in the thermal environment, we compare the two noise factors n_τ in Eq. (13) and n_d in Eq. (30). We have implicitly assumed in this paper that the two-mode squeezed state (quantum channel) generator is located in the middle point between the sending and receiving stations. The squeezed photons in the quantum channel, thus, travel only a half distance between the sending and receiving stations. Bearing it in mind, we find that

$$n_\tau \text{ (for time } t/2) - n_d \text{ (for time } t) = \bar{n}[1-\sqrt{1-T}]^2 + 1 - \sqrt{1-T}[1-\exp(-2s_{qc})]. \tag{31}$$

The right-hand side is semi-positive so that the noise given by teleportation is more than that by direct transmission. If we consider that this result is obtained for the case when the other operations including detection and unitary transformation in the teleportation protocol is perfect, we conclude that *the nonclassical field is more robust in direct transmission than in teleportation*. The reason is that the teleportation relies on quantum entanglement of the quantum channel. The quantum entanglement based on inter-mode coherence is much more fragile than the single-mode coherence. However, the quantum teleportation can be made more faithful by purification of the quantum channel while the direct transmission does not have that possibility.

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APPENDIX: POSITIVITY OF P FUNCTION AND SEPARABILITY FOR A GAUSSIAN STATE

A two-mode Gaussian state $\hat{\rho}$ of mode b and c is separable when it is represented by a statistical mixture of the direct-product states;

$$\hat{\rho} = \int d^2\beta \mathcal{P}(\beta)\hat{\rho}_b(\beta) \otimes \hat{\rho}_c(\beta) \tag{A1}$$

where $\hat{\rho}_{b,c}(\beta)$ are density matrices, respectively, for modes b and c, and $\mathcal{P}(\beta)$ is a probability density function with $\mathcal{P}(\beta) \geq 0$. The states of $\hat{\rho}_b(\beta)$ and $\hat{\rho}_c(\beta)$ can be nonclassical and do not have to have their P functions positive well-defined. However, because they are Gaussian, it is possible to transform them to assign positive well-defined P functions by local unitary transformations [26]. The separable condition, (A1), can then be written as

$$\begin{aligned}\hat{\rho}' = \int d^2\alpha_b \int d^2\alpha_c \int d^2\beta \mathcal{P}(\beta)P(\alpha_b;\beta)P(\alpha_c;\beta) \\ \times|\alpha_b\rangle\langle\alpha_b| \otimes |\alpha_c\rangle\langle\alpha_c|\end{aligned} \tag{A2}$$

where $P_b(\alpha_b;\beta)$ and $P_c(\alpha_c;\beta)$ are the P functions, respectively, for the fields of modes b and c after some local unitary operations. $\hat{\rho}'$ is for the two-mode Gaussian state after the local operations.

We want to prove in this appendix that if and only if when a two-mode Gaussian state is separable, a positive well-defined P function $P(\alpha_b, \alpha_c)$ is assigned to it after some local unitary transformations.

consider the sufficient condition. If a two-mode Gaussian state $\hat{\rho}$ is separable, it can be written as Eq. (A2) after some local operations. Both $P_b(\alpha_b;\beta)$ and $P_c(\alpha_c;\beta)$ are positive well-defined and $\mathcal{P}(\beta)$ is a probability density function so

$$\int d^2\beta \mathcal{P}(\beta)P(\alpha_b;\beta)P(\alpha_c;\beta) \tag{A3}$$

is a normalized positive function, which we can take as the positive well-defined P function $P(\alpha_b, \alpha_c)$. We have proved that if a two-mode Gaussian state is separable, it has a positive well-defined P function after some local unitary operations.

Now let us prove the necessary condition. If the locally-transformed two-mode Gaussian state is represented by a positive well-defined P function $P(\alpha_b, \alpha_c)$, the separable condition (A2) becomes

$$P(\alpha_b, \alpha_c) = \int d^2\beta \mathcal{P}(\beta)P_b(\alpha_b;\beta)P_c(\alpha_c;\beta). \tag{A4}$$

Further by some additional squeezing and rotation it is always possible to have the rotationally-symmetric variance $[\Delta\alpha_i(\phi)]^2$ for any angle ϕ. After these transformations, the positive well-defined P function $P(\alpha_b, \alpha_c)$ can be written as

$$\begin{aligned}P(\alpha_b, \alpha_c) = \mathcal{N} \exp\Big[- \sum_{i,j=b,c} \alpha_i N_{ij} \alpha_j^* \\ + \sum_{i=b,c} (\alpha_i \lambda_i^* + \alpha_i^* \lambda_i) \Big]\end{aligned} \tag{A5}$$

where $\mathcal{N}$ is the normalization constant, N_{ij} a Hermitian matrix, and λ_i a complex number. The linear terms of α_i are not considered because they do not affect the proof. In fact, they can always be removed by some local displacement operations. Eq. (A5) can be written as

$$P(\alpha_b, \alpha_c) = \frac{\mathrm{Det}N_{ij}}{\pi^2} \exp\left(- \sum_{i,j=b,c} \alpha_i N_{ij} \alpha_j^* \right) \tag{A6}$$

where $\mathrm{Det}N_{ij}$ is the determinant of the Hermitian matrix N_{ij}. To find an expression in the form of Eq. (A4), let us introduce an auxiliary field (β, β^*) enabling the function $P(\alpha_b, \alpha_c)$ to be represented by a Gaussian integral;

$$\begin{aligned}P(\alpha_b, \alpha_c) = \frac{\mathrm{Det}N_{ij}}{\pi^3} \int d^2\beta \exp\Big(- |\beta|^2 - E_b(\alpha_b, \beta) \\ - E_c(\alpha_c, \beta)\Big)\end{aligned} \tag{A7}$$

where

$$\begin{aligned}E_b(\alpha_b, \beta) = \left(N_{bb} + |N_{bc}|^2\right)|\alpha_b|^2 - \alpha_b N_{bc}\beta^* \\ - \alpha_b^* N_{bc}^* \beta\end{aligned} \tag{A8}$$

$$E_c(\alpha_c, \beta) = (N_{cc} + 1)|\alpha_c|^2 + \alpha_c\beta^* + \alpha_c^*\beta \tag{A9}$$

The integrand in Eq. (A7) can now be decomposed into three Gaussian functions each of which satisfies the normalization condition because

$$N_{ii} > 0 \quad \text{and} \quad \mathrm{Det}N_{ij} > 0 \tag{A10}$$

for positive well-defined $P(\alpha_b, \alpha_c)$ in Eq. (A5). Taking

$$\begin{aligned}P_b(\alpha_b;\beta) = \frac{M_b}{\pi} \exp\Big(- M_b|\alpha_b|^2 + \alpha_b N_{bc}\beta^* \\ + \alpha_b^* N_{bc}^* \beta - \frac{|N_{bc}|^2}{M_b}|\beta|^2\Big)\end{aligned} \tag{A11}$$

$$\begin{aligned}P_c(\alpha_c;\beta) = \frac{M_c}{\pi} \exp\Big(- M_c|\alpha_c|^2 - \alpha_c\beta^* - \alpha_c^*\beta \\ - \frac{1}{M_c}|\beta|^2\Big)\end{aligned} \tag{A12}$$

$$\mathcal{P}(\beta) = \frac{M_s}{\pi} \exp\left(-M_s|\beta|^2\right) \tag{A13}$$

where $M_b = N_{bb} + |N_{bc}|^2$, $M_c = N_{cc} + 1$, and $M_s = \mathrm{Det}N_{ij}/(M_b M_c)$, the P function is finally obtained in the form of Eq. (A4). It is clear that $\mathcal{P}(\beta)$ is the positive probability density function and the two-mode Gaussian state is separable if it can be transformed to have a positive well-defined P function by some local unitary operations.

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